Originally published in *Reference and Anaphoric Relations*, U. Egli & K. von Heusinger (ed.), Dordrecht: Kluwer, (1999) pp. 147-153. Pagination matched as closely as possible.

# Persistence, Polarity, and Plurality

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(A), (B), and (C) below are taken by many semanticists to be good working hypotheses:

(A) A sentence of the form *the As are Bs* is true (on its distributive reading) if, and only if, every *A* is *B* and there is more than one *A*. (Russell (1919), Chomsky (1975), Evans (1977), Neale (1990)).

(B) So-called "negative polarity" expressions like *ever* and *any* can occur only in "downward entailing" (↓) environments. (Ladusaw (1981), May (1985), Larson (1990)).

(C) Plural descriptions may contain negative polarity items, as in the following example, from May (1985):

But the conjunction of (A), (B), and (C) is inconsistent. Put simply, the problem is that if hypothesis (A) is correct, then neither *ever* nor *any* occurs in a ↓ environment in (1):

(1) The students who had ever read anything about phrenology attended Gall's lecture.

So by hypothesis (B) the string in (1) ungrammatical. In view of the empirical and methodological issues involved and the fact that many people I have talked to are reluctant to give up *any* of (A), (B), and (C), I shall set out the argument for inconsistency in detail and with some relevant background.

One way of thinking about Russell's Theory of Descriptions is as follows: Whereas the determiners *every* and *all* introduce universal quantifications and the determiners *a, an,* and *some* introduce existential quantifications, the determiner *the* simultaneously introduces *both*.[2] This comes through very

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1 Thanks to William Craig, Richard Larson, Peter Ludlow, Robert May, and Barry Smith for discussion.

2 Russell (1905, 1919, 1925).

clearly in Russell's formal and informal analyses of *the A is B* as given by (2) and (3), respectively:

(2) $\exists x(\forall y(Ay \leftrightarrow y{=}x) \cdot Bx)$

(3) there is at least one *A*, there is at most one *A*, and every *A* is *B*.

For certain philosophical purposes, it is useful to think of definite descriptions as complex *existential* phrases; for other purposes it is more useful to think of them as complex *universal* phrases. For example, if one focuses on the existential character of Russell's proposal it is easy to explain it to the novice by building upon Russell's proposal for *in*definite descriptions (phrases of the form *an A*) because a sentence of the form *an A is B* is analysed by Russell as $\exists x(Ax \cdot Bx)$. But as Chomsky (1975) observes, if one focuses on the universal character of the proposal the relationship between singular and plural definite descriptions comes more clearly into view. Chomsky suggests there is a just a cardinality difference between the truth condition of an utterance of a sentence containing a singular description and that of an utterance of the corresponding sentence containing the description in its plural form: *the A is B* is true if and only if all *A*s are *B*s and there is exactly one *A*; *the As are Bs* is true if and only if all *A*s are *B*s and there is more than one *A*. Or, in a notation that will be more useful to us in a moment:[3]

(4) *the A is B* is true iff $|\mathbf{A} - \mathbf{B}| = 0$ and $|\mathbf{A}| = 1$

(5) *the As are Bs* is true iff $|\mathbf{A} - \mathbf{B}| = 0$ and $|\mathbf{A}| > 1$.

So much, then, for hypothesis A, which captures a wealth of semantical data and inherits the positive semantical and ontological features of Russell's analysis of singular descriptions.[4]

Let us turn now to hypothesis (B). To understand this hypothesis and its application to sentences containing Russellian descriptions, we need to

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3 $\mathbf{X}$ is the set of things that are *X*; $\mathbf{X} - \mathbf{Y}$ is the set of things that are *X* but not *Y*; and $|\mathbf{X}|$ is the cardinality of the set of things that are *X*.

4 It should be stressed that the contents of (4) and (5) can be given by quantificational axioms of a broadly Tarskian nature that appeal to sequences. For 'the' with a singular complement, the following would suffice:

(i) $\forall s,k,\phi,\psi$ (*s* satisfies '[*the* $x_k$: $\phi$]' if and only if the sequence satisfying $\phi$ and differing from *s* at most in the *k*-th position also satisfies $\psi$).

The right-hand side of (i) should be interpreted as equivalent to "there is exactly one sequence satisfying $\phi$ and differing from *s* at most in the *k*-th position and every such sequence satisfies $\psi$" (for discussion, see Neale (1993)). For 'the' with a plural complement, replace 'sequence' by 'sequences' in the right hand side of (i) and interpret it as equivalent to "there is more than one sequence satisfying $\phi$ and differing from *s* at most in the *k*-th position and every such sequence satisfies $\psi$". The viability of a formal language containing restricted quantifiers (a) shows that the language of *Principia Mathematica* is not an essential ingredient of the Theory of Descriptions and (b) exposes the hollowness of objections to the theory that find fault with Russell's own formalism.

isolate four interesting monotonicity (↑↓) properties that determiners may have.[5]

(i) A determiner *D* is ↑1 ("one up")—"persistent" or "upward entailing in first position"—if and only if (6) is valid:

(6) If **D**(**A**, **B**) and **every**(**A**, **A′**) then **D**(**A′**, **B**).

E.g., *some*. Quick test: if *D* is ↑1 then (7) entails (8):

(7) *D* father(s) left
(8) *D* man/men left.

(ii) *D* is ↓1 ("one down")—"anti-persistent" or "downward entailing in first position"—if and only if (9) is valid:

(9) If **D**(**A**, **B**) and **every**(**A′**, **A**) then **D**(**A′**, **B**).

E.g., *every* and *no*. Quick test: If *D* is ↓1 then (8) entails (7).

(iii) *D* is ↑2 ("two up")—"monotone increasing" or "upward entailing in second position"—if and only if (10) is valid:

(10) If **D**(**A**, **B**) and **every**(**B**, **B′**) then **D**(**A**, **B′**).

E.g., *every* and *some*. Quick test: If *D* is ↑2 then (11) entails (8)

(11) *D* man/men left early
(8) *D* man/men left.

(iv) *D* is ↓2 ("two down")—"monotone decreasing" or "downward entailing in second position"—if and only if (12) is valid:

(12) If **D**(**A**, **B**) and **every**(**B′**, **B**) then **D**(**A**, **B′**).

E.g., *no* and *few*. Quick test: If *D* is ↓2 then (8) entails (11).

Now the reader can quickly verify that the ↑↓ properties of *some* and *no* are not sensitive to whether singular or plural complements are taken. For example, *some* is ↑1 and ↑2 in both (13) and (14):

(13) Some man left
(14) Some men left.

Similarly, *no* is ↓1 and ↓2) with both singular and plural complements; and *every* and *all* are both ↓1 and ↑2—we might, perhaps, think of *all* as the (syntactically) plural version of *every*, though nothing I shall say here turns on this.

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5 The general interest of these properties to semantics and syntax is discussed in detail by Barwise and Cooper (1981).

What of the ↑↓ properties of *the*? Intuitively, it is clear that on a Russellian analysis *the* is going to be ↑2 and $\not\uparrow\not\downarrow$1 (non-monotone, or flat, in first position). It will be ↑2 because *some* and *every* are both ↑2. That *the* is $\not\uparrow\not\downarrow$1 may seem less obvious, but it is really a consequence of the fact that *some* is ↑1 and *every* is ↓1. The easiest way to see that *the* is $\not\uparrow\not\downarrow$1 is to run the test given above with unpacked descriptions. (15) and (16) become (15′) and (16′) respectively:

(15) The father left
(16) The man left
(15′) There is exactly one father and every father left
(16′) There is exactly one man and every man left.

Neither (15′) nor (16′) entails the other, so on Russell's account neither (15) nor (16) entail one another. Thus *the* is $\not\uparrow\not\downarrow$1 when it takes a singular complement. Naturally enough, the situation is analogous when *the* takes a plural complement as we can see when (17) and (18) are unpacked as (17′) and (18′) respectively:

(17) The fathers left
(18) The men left
(17′) There are at least two fathers and every father left
(18′) There are at least two men and every man left.

Neither (17′) nor (18′) entails the other, so by hypothesis (A) neither (17) nor (18) entails the other. Thus *the* is also $\not\uparrow\not\downarrow$1 when it takes a plural complement (as expected, given the number symmetry observed for *some, every*, and *no*).

According to hypothesis (B), negative polarity items such as *ever* and *any* can only occur in ↓ environments. This hypothesis appears to explain the following data:[6]

(19) No man who has *ever* been to Boston has returned
(19′) No man who has been to Boston has *ever* returned
(20) Every man who has *ever* been to Boston has returned
(20′) * Every man who has been to Boston has *ever* returned
(21) * Some man who has *ever* been to Boston has returned
(21′) * Some man who has been to Boston has *ever* returned.

Since *no* is ↓1/↓2, *ever* may occur within either argument of the determiner; thus (19) and (19′) are both acceptable. Since *every* is ↑1/↓2, *ever* may occur only within its second argument; thus (20) is ruled out. Since *some* is ↑1/↑2, both (21) and (21′) are ruled out.[7]

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6 These data are taken almost *verbatim* from Larson (1990).

7 To summarise in this way is not to assume that determiners like *no, some, every*, etc. *must* be treated as "binary quantifiers" in the sense of Evans (1977) and Wiggins (1980). The summary above is perfectly compatible with the view that determiners are

(footnote continued next page)

Now if hypothesis (A) is correct, *the* is ↑↓1/↑2, and by hypothesis B the following are all ruled ungrammatical (a '?' in front of a string indicates that it is *sub judice*, that it is not entirely clear that it is ungrammatical):

(22) ? The man who has *ever* been to Boston has returned
(22′) * The man who has been to Boston has *ever* returned
(23) ? The men who have *ever* been to Boston have returned
(23′) * The men who have been to Boston have *ever* returned

But according to May (1985, pp. 25-26), string (23) is a grammatical sentence, and so is (1):

(1) The students who had ever read anything about phrenology attended Gall's lecture.

Let us put aside for the moment the question of whether or not (23) and (1) are grammatical and focus on the tension that their purported grammaticality creates for May's own theory. At pp. 8-9, May endorses an explicitly Russellian semantics for singular definite descriptions.[8] And on such an account, *the* is ↑↓1/↑2 when it takes a singular complement as May surely recognises. However, at pp. 10-11, he endorses hypothesis (B), which he supports with data similar to that used above. Thus, insofar as May agrees that on his own account *the* is ↑↓1 when it takes a singular complement, he is committed to the view that (22) is unacceptable. But at the same time he claims that (23) *is* acceptable. Thus May's position either gives rise to an inconsistency concerning *the* or else entails that *the* switches from ↑↓1 to ↓1 when it takes a plural complement; and this of course means giving up hypothesis (A), according to which *the* remains ↑↓1 when it takes a plural complement.

In correspondence, May has indicated to me that in order to preserve his data he is willing to accept the switch and ditch the hypothesis (A). But this seems to me like an exorbitant price in view of the fact that (i) there appears to be no other determiner whose ↑↓ properties are sensitive to whether its complement is singular or plural; and (ii) hypothesis (A) explains a wide range of linguistic data involving plural definite descriptions and plural unbound anaphoric pronouns. I suggest, then, that we look elsewhere for a solution to this little problem.

One idea would be simply to reject May's intuitions concerning the acceptability of strings (1) and (23), perhaps attributing an illusion of

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"unary quantifier-formers" that combine with nominal expressions to form restricted quantifiers as in (e.g.) Higginbotham and May (1981), and May (1985). The only argument I know for the view that determiners *must* be treated as binary quantifiers is due to Evans (1977) and is fallacious. For discussion, see Neale (1993).

8 As May puts it, his semantics "embeds the existence and uniqueness properties of definite descriptions, found invariantly under alternative scopes" (p. 9). See also Higginbotham and May (1981) pp. 68-69.

acceptability to performance factors of one form or another. On this view, hypotheses (A) and (B) remain intact.

A second idea would be to replace the hypothesis (B) by hypothesis (B′): negative polarity items may appear only in $\not\uparrow$ (non-$\uparrow$) environments (i.e. environments that are either $\not\uparrow\not\downarrow$ or $\downarrow$).[9] On such an account, along with (22) and (23) the following ought to be acceptable (again, '?' before a string is meant to indicate that it is *sub judice*):

(24) ? Exactly four men who have *ever* been to Boston have returned
(24′) ? Exactly four men who have been to Boston have *ever* returned
(25) ? The five men who have *ever* been to Boston have returned
(26) ? Most men who have *ever* been to Boston have returned.[10]

A third possibility, first suggested to me (but not necessarily endorsed) by Richard Larson, is that the phrases of the form *the Fs* in (1) and (23) are actually "disguised partitives," elliptical for phrases of the form *all of the Fs*. On such an account, the negative polarity item *ever* is said to occur in a $\downarrow$1 environment by virtue of occurring within the scope of *all*, which is $\downarrow$1. There are at least two ways of cashing out this idea. On one approach, *all of the*, *some of the*, *none of the*, etc. are complex determiners that (i) combine with a noun complex to form an NP, and (ii) inherit their $\uparrow\downarrow$ properties from *all*, *some*, and *no* respectively (rather than from *the*). On this account, the occurrences of *the* in (1) and (23) are then treated as elliptical for occurrences of a more complex (structured) determiner *all of the* (or *every one of the*).[11] Consequently, the negative polarity item *ever* occurs in a $\downarrow$ environment in (1) and (23) by virtue of lying within the first position of the determiner *all of the.*

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9 Or, intermediate between hypotheses (B) and (B′), hypothesis (B″): negative polarity items may only occur in $\not\uparrow\not\downarrow$1 or $\downarrow$ environments. Whatever the empirical merits of (B″), its asymmetry makes it unattractive to me.

10 My informants (philosophy and linguistics graduate students at Berkeley and London) tend to find these much better than (20′), (21), and (21′), which might suggest that the only valid generalisation is that negative polarity items are *fine* in $\downarrow$ environments, only *so-so* in $\not\uparrow\not\downarrow$ environments, and plain *bad* in $\uparrow$ environments.

11 The final part of this proposal may appear to have an *ad hoc* character; but perhaps it has more plausibility than initially meets the eye. First, as Lewis (1975) observes, explicit universal quantifications are sometimes implicit in a way that other quantifications are not. Thus (i) is standardly taken to be equivalent to (ii) and not to (iii) or (iv):

(i) If a man buys a donkey he beats it
(ii) If a man buys a donkey he always beats it
(iii) If a man buys a donkey he sometimes beats it
(iv) If a man buys a donkey he usually beats it

Second, appeals to ellipsis are independently needed to account for the semantic properties of so-called *incomplete* quantifiers introduced by $\downarrow$1 and $\not\uparrow\not\downarrow$1 determiners, i.e. non-persistent quantifiers (and perhaps even $\uparrow$1, determiners). This comes through particularly clearly with incomplete descriptions such as *the table*.

A second approach would be to treat *all of*, *some of*, *none of*, etc. as devices that combine with NPs of the form *the Fs* to create larger NPs. On this account, the negative polarity item *ever* occurs in a ↓ environment in (1) and (23) by virtue of lying within the first position of *all*.

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